SOLUTION OF THE HIGH TWIST EVOLUTION EQUATION IN THE DOUBLE LOGARITHMIC APPROXIMATION IN QCD

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Abstract

The evolution equation for high twist operators is solved in the Double Logarithmic Approximation for cylinder-type diagrams which dominate in the limit of large number of colors. The asymptotic behaviour of the parton correlation function is shown to be determined by the spectrum of the Ising model for a one-dimensional chain of spin-1/2 magnets.

1. The strong growth of the deep-inelastic structure function with decreasing Bjorken variable x_B leads to the necessity for taking into account the contribution of high twist operators since when $x_B \to 0$ the power suppression $1/Q^2$ can be compensated by the large logarithmic factors $\sim \ln 1/x_B$. Indeed, the moments of the structure function

$$M(j,Q^2) = \int_0^1 dx_B x_B^{j-1} x_B G(x_B,Q^2), \tag{1}$$

where Q^2 is a square of virtual photon momentum, can be expressed through a Wilson Operator Product Expansion in the form (see[1])

$$M(j,Q^2) = \sum_{N\geq 0} \frac{1}{Q^{2N}} C_{2N}(j,Q^2) \langle h \mid O^{(2N)} \mid h\rangle. \tag{2}$$

Here $C_{2N}(j,Q^2)$ is the coefficient function and $\langle h \mid O^{(2N)} \mid h\rangle$ denotes the expectation value of the local operator in a hadron state. Usually one can neglect the contribution of high twists (i.e. the terms with $N \geq 1$ in (2)) as they are suppressed by the powers of the large momentum square Q^2. However this is not the case for $x_B \to 0$. As is known from renormalization group arguments the coefficient function is expressed through anomalous dimension $\gamma_{2N}(j-1)$

$$C_{2N}(j,Q^2) \sim e^{\gamma_{2N}(j-1)\ln Q^2/\mu^2}, \tag{3}$$

where μ^2 is an infrared cut-off. The small-x_B behavior of the structure function is controlled by the singularity of γ_{2N} at $j = 1$. For the twist 2 case the anomalous dimension is given by the kernel of Gribov-Lipatov-Altarelli-Parisi (GLAP) equation in the gluon-gluon channel, and near $j = 1$ it has the form:

$$\gamma_2(j-1,Q^2) = \frac{N_c \alpha_S}{\pi(j-1)} \tag{4}$$

(N_c is the number of colors). The quark contributions are negligible here since they are not singular for $j \to 1$. Supposing the high twist anomalous dimension γ_{2N} to have the same singularity at $j = 1$ one can estimate the contribution of the twist $2N$ operator to the structure function for $x_B \to 0$ as

$$x_B G_{2N}(x_B,Q^2) \sim \left(\frac{\mu^2}{Q^2}\right)^N \exp \lambda_{2N} \sqrt{\ln Q^2/\mu^2 \ln 1/x_B}, \tag{5}$$

where the coefficient λ_{2N} is determined by the residue of γ_{2N} at $j = 1$ .This relation suggests existence of a kinematic region of the x_B, Q^2 variables where the power suppression is compensated by the logarithmic factors in the exponent. The line where the twist 2 and twist 4 contributions are of the same order is the boundary of this region, and in the interior of it the twist 4

becomes leading. There is an infinite set of similar domains where the twist 6 operators turn out to be dominant ones, then twist 8 and so on. The exact location and boundaries of these domains are determined by the coefficients λ_{2N} in Eq.(5).Inside these domains GLAP evolution equation becomes invalid and has to be replaced by the appropriate equation for high twists.

In paper [2] the specific contribution to the anomalous dimension γ_{2N} coming from the exchange of $2N$ non-interacting ladders (Pomerons) was found to be

$$\gamma_{2N}(\omega) = N\gamma_2(\tfrac{\omega}{N}), \quad \omega = j-1 \tag{6}$$

or

$$x_B G_{2N}(x_B, Q^2) \sim \left(\frac{\mu^2}{Q^2}\right)^N \exp N\sqrt{\frac{\alpha_S N_c}{4\pi}\ln Q^2/\mu^2 \ln 1/x_B} \tag{7}$$

that is the product of one-Pomeron asymptotics.

The calculations for twist 4 case performed in Refs.[3, 4] with the account of Pomerons interaction in Double Logarithmic Approximation (DLA) modified this result:

$$\gamma_4(\omega) = 2\gamma_2(\frac{\omega}{2})(1+\delta) = \frac{4N_c\alpha_S}{\pi\omega}(1+\delta) \tag{8}$$

with $\delta \simeq (N_c^2-1)^2 \approx 10^{-2}$. The corrections due to other states (color ladders) were estimated to be approximately of the same order. The δ value which indicates the deviation from Eq.(6) can be treated as a measure of the Reggeon interaction strenght. The problem how the Reggeon interaction modifies result (7) will be concidered below.

2. This paper will be based on the direct solution of the evolution equation for high twist operators in DLA.

The evolution equation for the high twist operators was derived in paper [5]. It generalizes the GLAP equation for quasipartonic operators which form the closed set of high twist operators allowing for the interpretation in terms of the parton model. Decomposing the momenta of virtual gluons k_i into Sudakov variables

$$k_i = \alpha_i q' + \beta_i p' + k_{\perp i}, \quad (p')^2 = (q')^2 = 0, \quad k_{\perp i}p' = k_{\perp i}q' = 0,$$

where p' is a hadron momentum, $q = q' - x_B p'$ is a photon momentum, $q^2 = -Q^2$, matrix elements of the twist N quasipartonic operators between parton states is expressed only through the β_i variables and has a general form

$$\rho^{a_1,\ldots,a_N}_{0,\mu_1,\ldots,\mu_N}(\beta_1,\ldots,\beta_N) \;=\; \Gamma^{a_1,\ldots,a_N}_{\mu_1,\ldots,\mu_N}\beta_1^{n_1}\cdots\beta_N^{n_N}, \tag{9}$$

where n_i are integer numbers and tensor Γ specifies the color and Lorentz structure of the operator. The variable β_i has a meaning of the fraction of the total hadron momentum carried by a parton.

In axial gauge $q'_\mu A_\mu = 0$ only the ladder-type diagrams contribute to the leading logarithms. For the twist N case they comprise the local operator vertex of the form (9) and N gluons in t -channel interacting through all possible s-channel gluons rungs. Very important property of Leading Logarithmic Approximation (LLA) is strict ordering of transverse momenta along the ladder graph, namely, the transverse virtualities decrease from the top of the diagram to the bottom. The range of integration in each ladder cell is such that transverse momentum above the rung is greater than the momentum below it. In other words the above momentum plays the role of ultraviolet cut-off for the below cell. Thus the Q^2 value, being the greatest momentum in the upper loop that is attached to the local operator, is the ultraviolet cut-off for the whole graph. The evolution equation for the quasipartonic operators is derived from the diagrams with pure longitudinal external gluons' momenta $p_i \simeq \beta_i p'$. Their sum determines the parton correlation function $\rho(Q^2, \beta_1, \cdots, \beta_N)$ the evolution equation for which is obtained by taking the derivative of the diagrams with respect to $\ln Q^2$. It has the form of the N-particle one dimensional equation with pairwise interaction in which the variable $\xi(Q^2) \equiv \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_S(k^2)}{4\pi}$ is an effective "time" ($\alpha_S(k^2)$ is the running coupling constant)

$$\frac{\partial}{\partial \xi}\rho(\xi, \beta_1, \cdots, \beta_N) \;=\; \sum_{i<j} \int d\beta'_i d\beta'_j \, \delta(\beta_i + \beta_j - \beta'_i - \beta'_j) \times \qquad (10)$$

$$\times \Phi(\beta_i, \beta_j \mid \beta'_i, \beta'_j)\, \rho(\xi, \beta_1, \cdots, \beta'_i, \cdots, \beta'_j, \cdots, \beta_N)$$

(to be more compact the color and Lorentz indexes are suppressed here). The kernel $\Phi(\beta_i, \beta_j \mid \beta'_i, \beta'_j)$ is determined by the logarithmic part of the one-loop integral over $k_\perp$ and α. The initial condition for the function $\rho(\xi)$ is given by the expression (9) that corresponds to the absence of loops. The equation (10) collects all the powers of $\alpha_S \ln Q^2/\mu^2$ where the infrared cut-off $\mu^2 \ll Q^2$ is the characteristic hadron scale.

The longitudinal momenta β_i are not in general ordered in LLA. But in DLA they also have to be ordered to provide a large logarithm for each ladder cell. The β_i variables increase from the values $\sim x_B$ at the local operator vertex to the order of unity ones in the lower part of a diagram. In such a kinematic the logarithmic divergencies that occur in every loop when $\beta'_i \to 0$ is cut from below by a longitudinal momentum in the upper cell. Thus DLA implies the loop integration in the evolution equation (10) to be limited by the condition

$$\beta'_i, \beta'_j \ll \beta_i, \beta_j \qquad (11)$$

which means that the momenta below the s-channel rung (β) and above it (β') are of different order of magnitude. The most singular contribution comes in Eq.(10) from the region where both β'_i and β'_j tend to zero. The

momentum conservation allows it only if

$$\beta_i + \beta_j \ll \beta_i, \beta_j \tag{12}$$

that is with logarithmic accuracy $\beta_i \approx -\beta_j$. Hereafter it is convenient to assume the momenta directed upward to be positive, directed downward to be negative.

In the logarithmic domain the kernel of the evolution equation can be easily obtained in helicity basis [5] by keeping the most singular in β' terms in gluon-gluon channel and integrating it over $k_\perp$ and α

$$\Phi^{a,b,c,d}_{\mu,\nu,\lambda,\sigma}(\beta_1,\beta_2 \mid \beta_1',\beta_2') \;=\; 2\delta_{\mu,\nu}\delta_{\lambda,\sigma} i f^{a,c,g} i f^{b,g,d}\, \beta_1 \delta(\beta_1-\beta_2)\,\frac{1}{\beta_1'\beta_2'}. \tag{13}$$

Here μ,ν,λ,σ are the two dimensional transverse indexes , $f^{a,c,g}$ are the structure constants of the $SU(N_c)$ group. The momenta β_1,β_2, are equal in DLA . They are positive but have the opposite directions, one of them is incoming from below the other is outgoing. The low-scale momenta β_1' and β_2' are not supposed to be equal in DLA since the momentum transfer from below $\beta_1-\beta_2$ is small only compared to the large momenta β_1 and β_2 but is of the same order as the low-scale ones. However β_1' and β_2' like β_1 and β_2 have the opposite directions, so the incoming from below momentum turns out into outgoing and vice versa. This property of the longitudinal momenta directions can be formulated as an "arrow rule"[6]. One can assign an arrow to each t-channel gluon line to indicate the momentum direction. The interaction occurs only between the lines with the opposite arrows while for the particular line the arrow direction does not change through all interactions. As a consequence all t-channel lines are divided into those incoming the local operator vertex and outgoing it. The violation of the "arrow rule" results in the loss of the longitudinal logarithm in the cell integration producing only small factor $\sim \alpha_S$. One should note here that the "arrow rule" will be effectively absorbed into restrictions imposed in the $N_c \to \infty$ limit where only the "neighbouring" gluons are allowed to interact.

It should be noted that the restriction only to the ladder type diagrams is valid in DLA provided the colorless in t-channel states are considered. Indeed, for these states only hard s-channel gluons carrying the longitudinal momentum fraction $\beta^{(s)} = \beta_i - \beta_i' \sim \beta_i$ are emitted. The logarithmic contributions from the soft non-ladder gluons emission cancel out when all the diagrams are summed up, since the soft particle probes the total color charge of the system which is zero for white operators [4].

The kernel (13) gives rise to DLA evolution equation

$$\frac{\partial}{\partial\xi}\rho^{a_1,\ldots,a_N}_{\mu_1,\ldots,\mu_N;\sigma_1,\ldots,\sigma_N}(\beta_1,\ldots,\beta_N) =$$

$$= \sum_{k<j} 2S_{\sigma_k,\sigma_j}\delta_{\mu_k,\mu_j}\delta_{\lambda_k,\lambda_j} if^{a_k,b_k,g} if^{g,b_j,a_j}\,\beta_k\delta(\beta_k-\beta_j)\times \qquad (14)$$

$$\times\int_0^{\beta_k}\frac{d\beta'_k}{\beta'_k}\int_0^{\beta_j}\frac{d\beta'_j}{\beta'_j}\,\rho^{a_1,\ldots,b_k,\ldots,b_j,\ldots,a_N}_{\mu_1,\ldots,\mu_N;\sigma_1,\ldots,\sigma_N}(\beta_1,\ldots,\beta'_k,\ldots,\beta'_j,\ldots,\beta_N).$$

Here all $\beta_i > 0$, the momentum direction and the "arrow rule" are specified by the indexes σ and coefficient S_{σ_k,σ_j}. The integration range in Eq.(14) is determined with logarithmic accuracy by condition (11). The initial condition is given by Eq.(9)

For twist-2 colorless operators the solution to Eq.(14) has the form

$$\rho^{a_1,a_2}_{\mu_1,\mu_2}(\xi,\beta_1,\beta_2) = \delta_{\mu_1,\mu_2}\delta^{a_1,a_2}\delta(\beta_1-\beta_2)f(\xi,\frac{x_B}{\beta}),$$

where function f obeys conventional GLAP evolution equation after substitution $\beta = 1$ (the structure function is considered to be dependent on the variable $-\frac{q^2}{2pq} = \frac{x_B}{\beta}$).

The color structure of the evolution equation is essentially simplified in the $N_c \to \infty$ limit. If in this case the color index is written as the pair of quark and antiquark ones $a = (i,\bar{\jmath})$ and gluons are drawn by double lines the interaction occurs only between the gluons containing the common "quark" line. The closed cycle arising when such t-channel gluon lines are connected produces a factor $\frac{1}{2}N_c$ while joining other lines gives an order of unity factor. There is a significant difference here with the twist-2 (Pomeron) solution for which the factor N_c is obtained after adding together two possible joining of two t-channel gluon lines. The gluons can be enumerated in such a way that the gluon with number n is allowed to interact only with the gluons $n-1$ and $n+1$. All the gluons can be represented to be lying on the cylinder in the color space and each of them interacts only with its neigbours. It is natural to assign to the neighbouring gluons opposite signs of β to ensure interaction between them. After this the color structure of the function ρ turns into itself under the action of the kernel of the evolution equation which takes the form

$$\frac{\partial}{\partial\xi}\rho_{\mu_1,\ldots,\mu_N}(\beta_1,\ldots,\beta_N) =$$

$$= \sum_n \delta_{\mu_n,\mu_{n+1}}\delta_{\mu'_n,\mu'_{n+1}}\,\beta_n\delta(\beta_n-\beta_{n+1})\times \qquad (15)$$

$$\times\int_0^{\beta_n}\frac{d\beta'_n}{\beta'_n}\int_0^{\beta_{n+1}}\frac{d\beta'_{n+1}}{\beta'_{n+1}}\,\rho_{\mu_1,\ldots,\mu'_n,\mu'_{n+1},\ldots,\mu_N}(\beta_1,\ldots,\beta'_n,\beta'_{n+1},\ldots,\beta_N).$$

Here ξ is redefined (for large N_c case) as

$$\xi(Q^2) = \int_{\mu^2}^{Q^2}\frac{dk^2}{k^2}\frac{N_c\alpha_S(k^2)}{4\pi}. \qquad (16)$$

3. There is a way to transform evolution equations (14) or (15) to the Schrödinger-type differential equation. The Laplace transformation

$$\rho(E) \;=\; \int_0^\infty d\xi\, e^{-E\xi}\rho(\xi) \tag{17}$$

brings the evolution equation to the stationary form, which in the logarithmic coordinates $x_i = \ln 1/\beta_i$ reads

$$E\rho_{r_1,\ldots,r_N}(x_1,\ldots,x_N) = \tag{18}$$

$$= -\sum_{i<k}\delta(x_i-x_k)\int_{x_i}^\infty dx_i'\int_{x_k}^\infty dx_k'\,A_{ik}\,\rho_{r_1,\ldots,r_N}(x_1,\ldots,x_i',\ldots,x_k',\ldots,x_N).$$

The arrow,Lorentz and color indexes are accumulated here in the index $r = \{\sigma,\mu,a\}$ and the matrix A_{ik} acting on the indexes at i,k locations is introduced

$$A_{ik} = 2S_{\sigma_i,\sigma_k}\delta_{\mu_i,\mu_k}\delta_{\mu_i',\mu_k'}if^{a_i,a_i',g}if^{g,a_k',a_k}. \tag{19}$$

Seeking the solution of Eq.(18) in the form

$$\rho_{r_1,\ldots,r_N}(x_1,\ldots,x_N) = \frac{\partial}{\partial x_1}\ldots\frac{\partial}{\partial x_N}\,\varphi_{r_1,\ldots,r_N}(x_1,\ldots,x_N) \tag{20}$$

the function φ obeys the equation

$$E\prod_{i=1}^N\frac{\partial}{\partial x_i}\,\varphi_{r_1,\ldots,r_N}(x_1,\ldots,x_N) = \tag{21}$$

$$= -\sum_{i<k}\delta(x_i-x_k)\prod_{\substack{j\neq i\\ j\neq k}}\frac{\partial}{\partial x_j}\,A_{ik}\,\varphi_{r_1,\ldots,r_N}(x_1,\ldots,x_N)$$

(φ is assumed to be vanishing at the upper limit). The coordinate space is separated into $N!$ sectors which are obtained from the fundamental one

$$x_1 < x_2 < \cdots < x_{N-1} < x_N \tag{22}$$

by the permutation of any two variables. The function φ satisfies inside these sectors the free equation

$$E\prod_{i=1}^N\frac{\partial}{\partial x_i}\,\varphi_{r_1,\ldots,r_N}(x_1,\ldots,x_N) = 0 \tag{23}$$

and has the form

$$\varphi_{r_1,\ldots,r_N}(x_1,\ldots,x_N) \;=\; e^{k_1x_1+\cdots+k_Nx_N} \tag{24}$$

or linear combination of such terms with some k_i equal to zero in each of them. At the boundary separating the two sectors the matching condition has to be fulfilled. Near, say, $x_1 = x_2$ hyper plane it reads

$$\prod_{j=3}^{N} \frac{\partial}{\partial x_j} E \frac{\partial}{\partial x_1} \frac{\partial}{\partial x_2} \varphi_{r_1,\ldots,r_N} = -\prod_{j=3}^{N} \frac{\partial}{\partial x_j} \delta(x_1 - x_2)\, \varphi_{r_1,\ldots,r_N}. \qquad (25)$$

Denoting the solution in the sectors $x_1 > x_2$ and $x_1 < x_2$ as ψ^{12} and ψ^{21} respectively, so that

$$\varphi = \theta(x_1 - x_2)\psi^{12}_{r_1,\ldots,r_N}(x_1,\ldots,x_N) + \theta(x_2 - x_1)\psi^{21}_{r_1,\ldots,r_N}(x_1,\ldots,x_N) \qquad (26)$$

the functions ψ are connected through the relations

$$\begin{array}{ll} 1) & \psi^{12} \mid_{x_1=x_2} = \psi^{21} \mid_{x_1=x_2} \\ 2) & E\left[\left(\frac{\partial}{\partial x_1}\psi^{21} - \frac{\partial}{\partial x_1}\psi^{12}\right) + \left(\frac{\partial}{\partial x_2}\psi^{12} - \frac{\partial}{\partial x_2}\psi^{21}\right)\right]\Big|_{x_1=x_2} = \\ & = -A_{12}\left(\psi^{12} + \psi^{21}\right)\big|_{x_1=x_2}. \end{array} \qquad (27)$$

Conditions (27) allow to establish the connection between the evolution equation and one-dimensional quantum mechanics of N particles described by the Schrödinger equation

$$\frac{E}{2}\sum_j \frac{\partial^2}{\partial x_j^2}\varphi_{r_1,\ldots,r_N} - \sum_{i<k}\delta(x_i - x_k)\, A_{ik}\, \varphi_{r_1,\ldots,r_N} \;=\; \mathcal{E}\varphi_{r_1,\ldots,r_N}. \qquad (28)$$

Indeed the function φ clearly has form (24) inside any sector of type (22) and satisfies conditions (27) at its boundary. Thus Eqs.(21) and (28) have the same wavefunction although there is not straightforward connection between the energies E and $\mathcal{E}$. For the large N_c case Eq.(28) can be rewritten in a chain-like form

$$\frac{E}{2}\sum_n \mathcal{H}_{n,n+1}\varphi_{\mu_1,\ldots,\mu_N}(x_1,\ldots,x_N) \;=\; \mathcal{E}\varphi_{\mu_1,\ldots,\mu_N}(x_1,\ldots,x_N), \qquad (29)$$

$$\mathcal{H}_{i,k} \;=\; \frac{1}{2}\left(\frac{\partial^2}{\partial x_i^2} \;+\; \frac{\partial^2}{\partial x_k^2}\right) \;+\; \delta(x_i - x_k)\delta_{\mu_i\mu_k}\delta_{\mu_i'\mu_k'}.$$

The correlation function ρ built upon function φ through (20) represents the sum of the terms with various number of δ-functions. Note that the term without δ-functions vanishes after some k_i are put to zero. Its absence is evident also from the structure of the equation (21).

4. Instead of the solution of evolution equation (15) or (29) the approach based on the direct summation of the ladder-type diagrams in the $N_c \to \infty$ limit will be adopted here. To begin with the formal solution to the evolution equation (10) can be written as

$$\rho(\xi) = e^{\Phi\xi}\rho_0 = \sum_{n=0}^{\infty}\frac{1}{n!}\,\xi^n\Phi^n\rho_0, \qquad (30)$$

where Φ is the kernel of the evolution equation. The term Φ^n acting on the initial state ρ_0 generates the ladder-type diagrams with n loops in β-space (with n integrals over β). To restore the proper expression (30) each β-loop has to be multiplied by variable ξ and the whole diagram has to be divided by $n!$. Introducing function

$$W(E) \,=\, \sum_{n=0}^{\infty} \frac{1}{E^n}\, \Phi^n \rho_0$$

the solution (30) is given by the formula

$$\rho(\xi) = \frac{1}{2\pi i} \oint \frac{dE}{E}\, e^{E\xi} W(E),$$

where the integration goes aroung $E = 0$ point in the counter-clock-wise direction. Function $W(E)$ obeys Bethe-Solpiter equation

$$W = \rho_0 \,+\, \frac{1}{E}\, \Phi W \tag{31}$$

describing the system of N t-channel particles with pairwise interaction between particles n and $n+1$

$$\Phi_{\mu,\nu,\lambda,\sigma}(\beta_1, \beta_2 \mid \beta_1', \beta_2') \,=\, \delta_{\mu,\nu}\delta_{\lambda,\sigma}\, \beta_1 \delta(\beta_1 - \beta_2)\, \frac{1}{\beta_1' \beta_2'}. \tag{32}$$

Equation (31) gives rise to the set of ladder-type planar diagrams in β-space. To sum it up they can be transformed to the form where the interaction between t-channel gluons is treated as an exchange of s -channel particles. Kernel (32) results effectively from the exchange of the particle whose emission is given by the expression (Fig.1):

$$T_{\mu\nu} \,=\, \frac{1}{\sqrt{E}}\, \theta(\beta - \beta') \frac{1}{\beta'}\, \sqrt{\beta}\, a^{\dagger}_{\mu\nu}(\beta), \tag{33}$$

where $a^{\dagger}_{\mu\nu}(\beta)$ ($a_{\lambda\sigma}(\beta)$) is a creation (annihilation) operator

$$\Big[a_{\lambda\sigma}(\beta)\,,\, a^{\dagger}_{\mu\nu}(\beta') \Big] = \delta_{\lambda\mu}\delta_{\sigma\nu}\delta(\beta - \beta').$$

Indeed, amplitude (33), being squared, reproduces evidently (32).

In a ladder-type diagram any t-channel line emits and absorbs arbitrary number of s-channel particles. For the particular t-channel line with number n there is associated an amplitude containing creation operators $a^{\dagger}_{n,\mu\nu}(\beta)$ and annihilation ones $a_{n-1,\lambda\sigma}(\beta)$. For instance, the amplitude shown in Fig.2 has a form

$$T_{\mu_1\mu_6} \,=\, E^{-\frac{5}{2}} \sum_{\mu_2,\cdots,\mu_5} \int dx \beta_1 \cdots d\beta_4 \times$$

$$\times\sqrt{x}\frac{a_{n,\mu_1\mu_2}(x)}{\beta_1}\,\sqrt{\beta_1}\frac{a^\dagger_{n-1,\mu_2\mu_3}(\beta_1)}{\beta_2}\,\sqrt{\beta_2}\frac{a_{n,\mu_3\mu_4}(\beta_2)}{\beta_3}\times$$

$$\times\sqrt{\beta_3}\frac{a_{n,\mu_4\mu_5}(\beta_3)}{\beta_4}\,\sqrt{\beta_4}\frac{a^\dagger_{n-1,\mu_5\mu_6}(\beta_4)}{y},$$

where the integration over the longitudinal momenta is submitted to the restriction

$$x > \beta_1 > \beta_2 > \beta_3 > \beta_4 > y. \tag{34}$$

Multiplying expressions of this type and applying Wick theorem to the product of the creation and annihilation operators one can reproduce all the ladder diagrams for the equation (31). Note that only the planar graphs will appear since the diagrams with the crosses between s-channel particles destroy β ordering (34). Assembling for a given t-channel line the terms with all possible emissions and absorbtions one gets the expression

$$T^{(n)}_{\mu\mu'}(x,y) \;=\; \sum_k E^{-\frac{k}{2}} \sum_{\mu_1,\cdots,\mu_k} \int_y^x d\beta_1 \int_y^{\beta_1} d\beta_2 \cdots \int_y^{\beta_{k-1}} d\beta_k \times$$

$$\times\,\frac{\sqrt{x}}{\beta_1}\,\Big(a_{n,\mu\mu_1}(x) + a^\dagger_{n-1,\mu\mu_1}(x)\Big)\,\frac{\sqrt{\beta_1}}{\beta_2}\,\Big(a_{n,\mu_1\mu_2}(\beta_1) + a^\dagger_{n-1,\mu_1\mu_2}(\beta_1)\Big)\times \tag{35}$$

$$\cdots\times\,\frac{\sqrt{\beta_k}}{y}\,\Big(a_{n,\mu_k\mu'}(\beta_k) + a^\dagger_{n-1,\mu_k\mu'}(\beta_k)\Big)\,.$$

Here x and y are the momenta of lower and upper points of the t-channel line (incoming and outgoing momenta, see Fig.2), μ and μ' are their polarization indexes. Introducing 2×2 matrix $A_n(\beta)$

$$A_{n,\mu\nu}(\beta) \;=\; a_{n,\mu\nu}(\beta) + a^\dagger_{n-1,\mu\nu}(\beta)$$

Eq.(35) can be rewritten as

$$T^{(n)}_{\mu\mu'}(x,y) =$$

$$= \sum_k \left(\frac{1}{\sqrt{E}}\,\frac{\sqrt{x}}{y}A_n(x)\int_y^x d\beta_1\int_y^{\beta_1} d\beta_2\cdots\int_y^{\beta_{k-1}} d\beta_k\,\frac{A_n(\beta_1)}{\sqrt{E\beta_1}}\cdots\frac{A_n(\beta_k)}{\sqrt{E\beta_k}}\right)_{\mu\mu'} =$$

$$= \left(\frac{\sqrt{x}}{x_B}\,A_n(x)\,P\exp\left\{\int_y^x d\beta\,\frac{A_n(\beta)}{\sqrt{E\beta}}\right\}\right)_{\mu\mu'}. \tag{36}$$

Here symbol P means ordering of $A_n(\beta)$ matrixes according which the matrix with larger argument stands to the left from the matrix with smaller one. An equivalent form of the latter expression is

$$T^{(n)}_{\mu\mu'}(x,x_B) = \sqrt{E}\frac{x}{y}\,\frac{\partial}{\partial x}\left(P\exp\left\{\int_y^x d\beta\,\frac{A_n(\beta)}{\sqrt{E\beta}}\right\}\right)_{\mu\mu'}. \tag{37}$$

Now the problem reduces formally to the evaluation of the correlator

$$K_{\mu_1,\mu'_1,\ldots,\mu_N,\mu'_N}(x_1,y_1,\ldots,x_N,y_N) =$$

$$= \left\langle 0 \left| T^{(1)}_{\mu_1\mu'_1}(x_1,y_1) T^{(2)}_{\mu_2\mu'_2}(x_2,y_2)\cdots T^{(N)}_{\mu_N\mu'_N}(x_N,y_N) \right| 0 \right\rangle, \tag{38}$$

where $|0\rangle$ is the vacuum of operators $a_1, a_2, \ldots, a_N$. Indeed, only Wick pairing terms survive after vacuum averaging, therefore correlator (38) includes all the ladder diagrams for Bethe-Solpiter equation (31), whose solution is expressed through it as

$$W_{\mu_1,\ldots,\mu_N}(x_1,\ldots,x_N) =$$

$$= \sum_{\mu'} \int dy_1 \cdots dy_N \, K_{\mu_1,\mu'_1,\ldots,\mu_N,\mu'_N}(x_1,y_1,\ldots,x_N,y_N) \rho_{0,\mu'_1,\ldots,\mu'_N}(y_1,\ldots,y_N).$$

For the cylinder topology case one has to replace in (38) $a_0^\dagger \to a_N$ in the first term and $a_N \to a_N^\dagger$ in the last one. It is needed to ensure Wick pairing between t-channel lines 1 and N to reproduce their interaction on the cylinder.

If it were not for the matrix structure of $T^{(n)}$ the correlator would be easily calculated using the relation

$$\exp\{\int d\beta \, U(\beta)\, a_n(\beta)\} \, \exp\{\int d\beta \, V(\beta)\, a_n^\dagger(\beta)\} = \tag{39}$$

$$= \exp\{\int d\beta \, V(\beta)\, a_n^\dagger(\beta)\} \, \exp\{\int d\beta \, U(\beta)\, a_n(\beta)\} \, \exp\{\int d\beta \, U(\beta) V(\beta)\}$$

which is valid for arbitrary functions $U(\beta)$ and $V(\beta)$. Indeed, in that case the correlator would be of the form

$$K(x_1,y_1,\ldots,x_N,y_N) = E^{\frac{N}{2}} \frac{x_1}{y_1} \cdots \frac{x_N}{y_N} \frac{\partial}{\partial x_1} \cdots \frac{\partial}{\partial x_N} \varphi(x_1,y_1,\ldots,x_N,y_N),$$

where

$$\varphi(x_1,y_1\ldots,x_N,y_N) =$$

$$= \left\langle 0 \left| \exp\left\{ \int_{y_1}^{x_1} \frac{d\beta}{\sqrt{E\beta}} a_1(\beta) \right\} \exp\left\{ \int_{y_1}^{x_1} \frac{d\beta}{\sqrt{E\beta}} a_n(\beta) \right\} \times \right.\right. \tag{40}$$

$$\left.\left. \times \exp\left\{ \int_{y_2}^{x_2} \frac{d\beta}{\sqrt{E\beta}} a_2(\beta) \right\} \exp\left\{ \int_{y_2}^{x_2} \frac{d\beta}{\sqrt{E\beta}} a_1^\dagger(\beta) \right\} \cdots \right| 0 \right\rangle.$$

The evaluation of this expression can proceed as follows. To begin with one can get rid off the fourth exponent in (40) moving it to the left through third and second ones, commuting it through relation (39) with the first one and taking into account that $\langle 0|a_1^\dagger(\beta) = 0$. Then the same can be done with the other exponents moving the terms with creation operators to the left and

with annihilation ones to the right. After a sequence of such manipulations the final result will be

$$\varphi(x_1, y_1, \ldots, x_N, y_N) = \tag{41}$$

$$= \exp\left\{\int_{y_{12}}^{x_{12}} \frac{d\beta}{E\beta} + \int_{y_{23}}^{x_{23}} \frac{d\beta}{E\beta} + \cdots + \int_{x_{N-1,N}}^{x_{N-1,N}} \frac{d\beta}{E\beta} + \int_{x_{N,1}}^{x_{N,1}} \frac{d\beta}{E\beta}\right\},$$

where the integrations limits $x_{ik} = min\{x_i, x_k\}$ and $y_{ik} = max\{y_i, y_k\}$ arise after substitution in relation (39) $U_i(\beta) = V_i(\beta) = \frac{1}{\sqrt{E\beta}}\,\theta(x_i - \beta)\theta(\beta - y_i)$. Differentiating these terms produces pre-exponent factors $\delta(x_{i-1} - x_i)$. Their presence in the correlation function has the origin in the evolution equation structure (see Eqs.(20) and (26)). In DLA framework all x_i in the exponents have to be taken to be the same and all $y_i \simeq x_B$. Then the leading behaviour of the correlator will be

$$\left\langle 0 \left| T^{(1)}(x, x_B) \cdots T^{(N)}(x, x_B) \right| 0 \right\rangle \sim \exp\{N\,\frac{1}{E}\ln\frac{x}{x_B}\}. \tag{42}$$

5. The matrix structure of (36) prevents to apply relation (39) immediatly. To avoid this difficulty P-exponent in (36) can be represented through the functional integral. This expression is derived in two steps. At first the β interval is divided into large number N of small intervals $\Delta\beta$:

$$P\,\exp\left\{\int_{x_B}^{x} \frac{d\beta}{\sqrt{E\beta}}\,A(\beta)\right\} \simeq \tag{43}$$

$$\simeq \left(I + \frac{\Delta\beta}{\sqrt{E\beta_N}}\,A(\beta_N)\right)\left(I + \frac{\Delta\beta}{\sqrt{E\beta_{N-1}}}\,A(\beta_{N-1})\right)\cdots\left(I + \frac{\Delta\beta}{\sqrt{E\beta_0}}\,A(\beta_0)\right)$$

$$\beta_N = x \quad \beta_0 = y,$$

the accuracy being the better the smaller value $\Delta\beta$ is taken. At the second step the following expression for the unit 2×2 matrix is inserted between the terms in the product (43)

$$c\int dz^* dz\,\delta(z^* z - 1)\,z_\lambda z^*_\sigma = \delta_{\lambda\sigma}.$$

Here c is a normalization constant and the integration is carried out over the components of two dimensional complex vector z,

$$\begin{aligned} dz^* dz &= dz^*_2 dz_2\,dz^*_1 dz_1 \\ \delta(z^* z - 1) &= \delta(z^*_2 z_2 + z^*_1 z_1 - 1). \end{aligned}$$

It gives

$$P\,\exp\left\{\int_{x_B}^{x} \frac{d\beta}{\sqrt{E\beta}}\,A(\beta)\right\}_{\mu\mu'} \simeq \int \prod_{k=0}^{N+1} dz^*_k dz_k \delta(z^*_k z_k - 1)\,z_{N+1,\mu} z^*_{0,\mu'} \times$$

$$\times \prod_{k=0}^{N} z_{k+1}^{*} \left(I + \frac{\Delta\beta}{\sqrt{E\beta_k}} A(\beta_k) \right) z_k. \tag{44}$$

Denoting $z_k = z(\beta_k)$ and taking into account that for $|z_k| = 1$

$$z_{k+1}^{*} z_k \;=\; 1 \;+\; (z_{k+1}^{*} - z_k^{*}) z_k$$

the product in (44) takes the form

$$\prod_{k=0}^{N} \left(1 \;+\; \dot{z}^{*}(\beta) z(\beta) \Delta\beta \;+\; z^{*}(\beta + \Delta\beta) \frac{\Delta\beta}{\sqrt{E\beta_k}} A(\beta) z(\beta) \right) \simeq$$

$$\simeq \; \exp \int_{x_B}^{x} (\dot{z}^{*} z + z^{*} \frac{\Delta\beta}{\sqrt{E\beta}} A\, z)\, d\beta,$$

where dot means the derivative over β. As a result the amplitude is given by the functional integral over the complex vector field $z(\beta)$:

$$T_{\mu\mu'}^{(n)}(x,y) \;=\; \sqrt{E}\, \frac{x}{y}\, \frac{\partial}{\partial x}\, P_{\mu\mu'}^{(n)}(x,y),$$

$$P_{\mu\mu'}^{(n)}(x,y) = c' \int Dz_n^{*} Dz_n \delta(z_n^{*} z_n - 1)\, z_{n,\mu}(x) z_{n,\mu'}^{*}(y) \times$$

$$\times \exp \left\{ \int_y^x d\beta \left[\dot{z}_n^{*} z_n \;+\; z_n^{*} \frac{A_n(\beta)}{\sqrt{E\beta}} z_n \right] \right\}. \tag{45}$$

Here c' is the normalization constant and

$$Dz_n^{*} Dz_n \delta(z_n^{*} z_n - 1) \;=\; \prod_{\beta} dz_n^{*}(\beta) dz_n(\beta) \delta(z_n^{*}(\beta) z_n(\beta) - 1). \tag{46}$$

Instead of an explicit δ-function in measure (46) one can rewrite the integral as

$$P_{\mu\mu'}^{(n)}(x,y) = c'' \int D\sigma_n \int Dz_n^{*} Dz_n \times$$

$$\times \exp \left\{ \int_y^x d\beta \left[\dot{z}_n^{*} z_n + i\sigma_n (z_n^{*} z_n - 1) + z_n^{*} \frac{A_n(\beta)}{\sqrt{E\beta}} z_n) \right] \right\}, \tag{47}$$

where auxiliary field $\sigma_n(\beta)$ is introduced. The functional representation (45) or (47) has to be substituted in correlator (38). Averaging of exponential factors over vacuum state can again be done by making use of formula (39) taking

$$U_{n,i}(\beta) a_{n,i}(\beta) \;=\; \frac{1}{E\beta}\, z_{n+1,\mu}^{*}(\beta) a_{n,\mu\nu}(\beta) z_{n+1,\nu}(\beta)$$

$$V_{n,i}(\beta) a_{n,i}^{\dagger}(\beta) \;=\; \frac{1}{E\beta}\, z_{n,\mu}^{*}(\beta) a_{n,\mu\nu}^{\dagger}(\beta) z_{n,\nu}(\beta).$$

It results into expression

$$K_{\mu_1,\mu'_1,\ldots,\mu_N,\mu'_N}(x_1,y_1,\ldots,x_N,y_N) = \int \prod_n Dz_n^* Dz_n \delta(z_n^* z_n - 1)\, z_{n,\mu_n}(x_n) \times$$
$$\times z^*_{n,\mu'_n}(y_n)\, F(z_n^*, z_n)\, \exp\left\{\sum_n \left[\int_{y_n}^{x_n} d\beta \dot{z}_n^* z_n \,+\, \int_{y_{n,n+1}}^{x_{n,n+1}} d\beta \frac{1}{E\beta}\, h_{n,n+1},\right]\right\} \quad (48)$$
$$h_{ik} \,=\, z^*_{i,\mu} z^*_{k,\mu} z_{i,\nu} z_{k,\nu},$$

where pre-exponential factor $F(z^*, z)$ arises after differentiating the exponent over x_i variables:

$$F(z^*, z) = \prod_i{}' [x_i z_i^*(x_i) z_i(x_i) + h_{i-1,i}\theta(x_{i-1} - x_i) + h_{i,i+1}\theta(x_{i+1} - x_i)]. \quad (49)$$

Prime here denotes that after multiplication one has to replace

$$(h_{ik}\theta(x_i - x_k))^2 \to h_{ik}\, x_i \delta(x_i - x_k).$$

Within DLA accuracy all y_i in the exponent have to be taken to be of the order of x_B while all $x_i \simeq x \sim 1$. After that integral (48)can be treated as a functional integral for a one-dimensional quantum mechanical chain.

Let $c^\dagger_{n,\lambda}$ and $c_{n,\lambda}$ be the creation and annihilation operators of the particles labelled by the index λ in the chain site n and let the dynamics of the system is described by the hamiltonian

$$H \,=\, \sum_n c^\dagger_{n+1,\mu} c^\dagger_{n,\mu}\, c_{n+1,\nu} c_{n,\nu}$$

which commutes with the operator of the number of particles in site n

$$N_n \,=\, \sum_{\lambda=1}^{2} c^\dagger_{n,\lambda} c_{n,\lambda}.$$

Writing δ-functions in the measure of Eq.(48) through additional integrals over functions $\sigma(\beta)$ the integral over z^*, z variables can be considered as a functional integral in the holomorphic representation written for the matrix element

$$M = \quad (50)$$
$$\left\langle 0_c \left| \prod_m c_{m,\mu_m} F(c^\dagger, c)\, \exp\left\{\int_{x_B}^{x} d\beta [\sum_n i\sigma_n(\beta)(N_n - 1) \,+\, \frac{1}{E\beta} H]\right\} \prod_n c^\dagger_{n,\mu'_n} \right| 0_c \right\rangle,$$

where $|0_c\rangle$ is the vacuum state with respect to operators $c_{n,\lambda}$. In the holomorphic representation operators $c^\dagger_{n,\lambda}$ and $c_{n,\lambda}$ are replaced by c-number functions $z^*_{n,\lambda}(\beta)$ and $z_{n,\lambda}(\beta)$ over which the integration is performed (see e.g. [8]). One

can easily check it up by dividing the "time" interval $[x_B, x]$ into large number of small ones and repeating step by step the derivation of the formula (45) inserting between the parentheses the unit operator

$$I \;=\; \int \prod_{n,\lambda} dz^*_{n,\lambda} dz_{n,\lambda}\, e^{-\sum_{n,\lambda} z^*_{n,\lambda} z_{n,\lambda}}\, |Z\rangle \langle Z|\,, \tag{51}$$

where the state

$$|Z\rangle \;=\; \exp\left\{ \sum_{n,\lambda} z_{n,\lambda} c^\dagger_{n,\lambda} \right\} |0_c\rangle$$

has the properties

$$\begin{gathered} c_{n,\lambda}\,|Z\rangle \;=\; z_{n,\lambda}\,|Z\rangle \\ \langle Z_2\,|\,Z_1\rangle \;=\; e^{-z^*_2 z_1} \\ \left\langle Z_2 \left| H(c^\dagger, c) \right| Z_1 \right\rangle \;=\; H(z^*_2, z_1) e^{-z^*_2 z_1} \end{gathered}$$

and operator H is supposed to be normal ordered.

The functional integral over $\sigma(\beta)$ gives $N_n = 1$ in (50), i.e. the nonvanishing contribution to the matrix element comes only from the states where there is only one particle in each chain site n. Acting on these states hamiltonian H transforms them into themselves since it does not change the total number of particles in the site. Finally, correlator (48) is expressed through the matrix element in the kinematic where $x_i \simeq x$, $y_i \simeq x_B$ by the formula

$$\begin{gathered} K_{\mu_1,\mu'_1,\ldots,\mu_N,\mu'_N}(x_1, \ldots, x_N) = \\ =< 0_c| \prod_m c_{m,\mu_m} F(c^\dagger, c)\, e^{\frac{1}{E} \ln \frac{x}{x_B} H} \prod_n c_{n,\mu'_n} |0_c > . \end{gathered} \tag{52}$$

The powers of $z^*_i(x_i) z_i(x_i)$ in pre-exponential factor $F(z^*, z)$ (49) do not contribute to functional integral (48) (one can treat them as a result of insertion expression (51) between unit operators) therefore

$$F(c^*, c) \;=\; N \left\{ \prod_i{}' [H_{i-1,i} \theta(x_{i-1} - x_i) + H_{i,i+1} \theta(x_{i+1} - x_i)] \right\}, \tag{53}$$

where

$$\begin{aligned} H_{ik} \;&=\; c^\dagger_{i,\mu} c^\dagger_{k,\mu} c_{i,\nu} c_{k,\nu}, \\ H \;&=\; \sum_n H_{n,n+1}, \end{aligned} \tag{54}$$

prime has the same meaning as in Eq.(49) and N means the normal ordering.

Introducing helicity basis $e^\pm = \frac{1}{\sqrt{2}}(1, \pm i)$ the two-body operators H_{ik} act on the chain wavefunction ψ as

$$\begin{gathered} (H_{ik}\psi)_{+-} = (\psi_{+-} + \psi_{-+}) \\ (H_{ik}\psi)_{-+} = (\psi_{-+} + \psi_{+-}) \\ (H_{ik}\psi)_{++} = (H_{ik}\psi)_{--} = 0, \end{gathered} \tag{55}$$

where all indexes except those at i, k positions are omitted. Introducing for each chain site an effective spin-$\frac{1}{2}$ variable s_z and treating the positive helicity as $s_z = \frac{1}{2}$ state and the negative one as $s_z = -\frac{1}{2}$ the term H_{ik} takes a form

$$H_{ik} \,=\, \left(S^i_+ S^k_- \,+\, S^i_- S^k_+\right) \,-\, \left(S^i_z S^k_z \,-\, \frac{1}{4}\right),$$

where $S_{+,-,z}$ are the spin operators. Thus the total chain hamiltonian (54) turns out to be the hamiltonian of Ising model for one-dimensional magnets (for closed chain).

However there is an uncertainity in this way of proceeding, namely, a general normalization constant that usually remains to be unknown in the functional integral approach. As a consequence hamiltonian H in (38) is determined up to constant E_0 which leads to the shift of the spectrum. To find E_0 one has to compare the result with a case where the answer is known since E_0 depends only on the way the functional integral is built up rather than a particular structure of the interaction. To this purpose the simple kernel

$$\Phi_{\mu,\nu,\lambda,\sigma}(\beta_1, \beta_2 \mid \beta'_1, \beta'_2) \,=\, \delta_{\mu,\lambda}\delta_{\nu,\sigma}\,\beta_1\delta(\beta_1 - \beta_2)\,\frac{1}{\beta'_1\beta'_2}.$$

can be taken instead of (32). This kernel is diagonal in polarization indexes, therefore the solution is given by expression (42). From the other hand it can be obtained through all functional integral machinery described above that leads to the quantum chain with hamiltonian

$$H \,=\, \sum_n c^\dagger_{n+1,\mu} c_{n+1,\mu}\, c^\dagger_{n,\nu} c_{n,\nu}.$$

Due to the restriction $N_n = 1$

$$H \,=\, \sum_n 1 \,=\, N$$

and the asymptotic behaviour of the matrix element is the same as correlator (42) has. Thus one can conclude that energy shift $E_0 = 0$.

5. The solution of the Ising model is well known. After substituting the wavefunction of the Schrödinger equation

$$H\,\psi^{(N)}_m \,=\, \varepsilon^{(N)}_m\,\psi^{(N)}_m$$

the asymptotic behaviour of matrix element (52) is defined by the maximal energy $\varepsilon^{(N)}_{max}$. Then

$$W(E) \,\sim\, \exp\left\{\frac{1}{E}\,\varepsilon^{(N)}_{max}\,\ln\frac{x}{x_B}\right\}$$

and

$$\rho(\xi) \,\sim\, \exp\sqrt{4\varepsilon^{(N)}_{max}\,\xi\ln\frac{x}{x_B}}.$$

In the Ising model the zero-energy state (vacuum) is the state where all spins have the same orientation (e.g. when all they downward directed). An excited state with m turned-off spins is treated as the state with m quasi-particles ($m < N$) which are characterized by momenta k_i. The excitation energy is given then by the formula

$$\varepsilon_m^{(N)} = \sum_{i=1}^{m} (\cos k_i + 1), \tag{56}$$

where k_i (or, more exactly, e^{ik_i}) values are determined (for the closed chain case) by the algebraic equation of high order (see e.g. [7]) so there is not an analityc expression for them. Nevertheless Eq.(56) shows that $\varepsilon_{max}^{(N)} < 2(N-1)$ (the state with $m = N$ obviously has zero energy). For large N one can expect the maximum energy for $m \simeq \frac{N}{2}$ which leads to $\varepsilon_{max}^{(N)} = sN$, where $s \sim 1$. Neglecting α_S running it gives for the structure function

$$x_B G_{2N}(x_B, Q^2) \sim \left(\frac{\mu^2}{Q^2}\right)^N \exp \sqrt{2sN \, \frac{\alpha_S N_c}{\pi} \, \ln Q^2/\mu^2 \ln 1/x_B}.$$

Thus the contribution of a single $2N$-particle cylinder is smaller than N Pomerons one (when $N_c \to \infty$ Pomeron can be treated as a two-particle cylinder). Qualitative agreement of this asymptotics with formula (42) shows the incorporation of gluon polarization does not change the behaviour of the structure function. This is a consequense of small number gluons interactions allowed by cylinder topology. The stronger N-dependence is to be expected for $\frac{1}{N_c}$ corrections. Therefore the investigation of the high twist evolution equation for finite N_c is of great interest.

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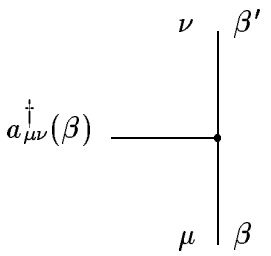

Fig.1. The effective amplitude.

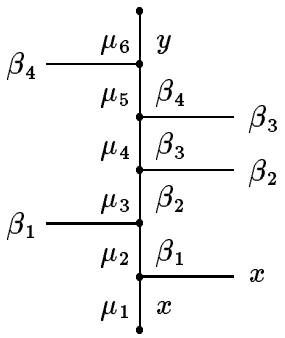

Fig.2. An example of t-line amplitude.